## 632

The publisher's errata for textbook pages 632 and 633 are themselves errata. The example as originally printed in the textbook is essentially correct. However, some of the example is awkwardly presented with various units omitted. The presentation below shows more of the algebra and shows the units more consistently.

## Example 9.4b

 M ore complete algebra and units more consistently represented.In this example we consider the design of the current source that supplies the bias current of a MOS differential amplifier. Let it be required to achieve a CMRR of 100 dB and assume that the only source of mismatch between $Q_{1}$ and $Q_{2}$ is a $2 \%$ mismatch in their $W / L$ ratios. Let $I=200 \mu \mathrm{~A}$ and assume that all transistors are to be operated at $V_{O V}=0.2 \mathrm{~V}$. For the $0.18 \mu \mathrm{~m}$ CMOS fabrication process available, $V_{A}^{\prime}=5 \mathrm{~V} / \mu \mathrm{m}$. If a simple current source is utilized for $I$, what channel length is required? If a cascode current source is utilized, what channel length is needed for the two transistors in cascode?

## Solution

A mismatch in $W / L$ results in a $g_{m}$ mismatch that can be found from the expression of $g_{m}$. The expression on the next line can be derived from Eq (7.41) on Page 388 and Eq (5.11) on Page 254.

$$
g_{m}=\sqrt{2\left(\mu_{n} C_{o x}\right)\left(\frac{W}{L}\right) I_{D}}
$$

Define $\left(\frac{W}{L}\right)_{\max }=1.02\left(\frac{W}{L}\right)$, a $2 \%$ increase. Similarly define $g_{m_{\max }}=g_{m}+\Delta g_{m}$

$$
g_{m_{\max }}=\sqrt{2\left(\mu_{n} C_{o x}\right)\left(\frac{W}{L}\right)_{\max } I_{D}}=\sqrt{2\left(\mu_{n} C_{o x}\right)(1.02)\left(\frac{W}{L}\right)_{\max } I_{D}}=\sqrt{1.02} g_{m}=1.01 g_{m}
$$

It is thus shown that an error of $2 \%$ in $W / L$ will result in an error in $g_{m}$ of $1 \%$. That is, the $2 \%$ mismatch in the $W / L$ ratios of $Q_{1}$ and $Q_{2}$ will result in a $1 \%$ mismatch in their $g_{m}$ values. The resulting CMRR can be found from Eq. (9.88), repeated here:

$$
C M R R=\left(2 g_{m} R_{S S}\right) /\left(\frac{\Delta g_{m}}{g_{m}}\right)
$$

Now, a 100 dB CMRR corresponds to a ratio of $10^{5}$; thus,

$$
10^{5}=\left(2 g_{m} R_{S S}\right) / 0.01
$$

The value of $g_{m}$ can be found from

$$
g_{m}=\frac{2 I_{D}}{V_{O V}}=\frac{2(I / 2)}{V_{O V}}=\frac{2(200 \mu \mathrm{~A}) / 2}{(0.2 \mathrm{~V})}=(1000 \mu \mathrm{~A} / \mathrm{V})=1 \mathrm{~mA} / \mathrm{V}
$$

Solving Eq. (9.90b) for $R_{S S}$ gives

$$
R_{S S}=\operatorname{CMRR}\left(\frac{\Delta g_{m}}{g_{m}}\right) /\left(2 g_{m}\right)=\left[10^{5}(0.01)\right] /[2(1 \mathrm{~mA} / \mathrm{V})]=1000 /(2 \mathrm{~mA} / \mathrm{V})=500 \mathrm{k} \Omega
$$

Now if the current source is implemented with a single transistor, its $r_{o}$ must be

$$
r_{o}=R_{S S}=500 \mathrm{k} \Omega
$$

Thus,

$$
\frac{V_{A}}{I}=500 \mathrm{k} \Omega
$$

Substituting $I=200 \mu \mathrm{~A}=0.2 \mathrm{~mA}$, we find the required value of $V_{A}$ as

$$
V_{A}=I r_{o}=(0.200 \mathrm{~mA})(500 \mathrm{k} \Omega)=100 \mathrm{~V}
$$

Since $V_{A}=V_{A}^{\prime} L=(5 \mathrm{~V} / \mu \mathrm{m}) L$, the required value of $L$ will be

$$
L=V_{A} / V_{A}^{\prime}=(100 \mathrm{~V}) /(5 \mathrm{~V} / \mu \mathrm{m})=20 \mu \mathrm{~m}
$$

which is very large!
Using a cascode current source, we have

$$
R_{S S}=\left(g_{m} r_{o}\right) r_{o} \quad \text { or, solving for } r_{o}, \quad r_{o}=\sqrt{R_{S S} / g_{m}}
$$

where

$$
g_{m}=\frac{2 I}{V_{O V}}=\frac{2(200 \mu \mathrm{~A})}{(0.2 \mathrm{~V})}=2 \mathrm{~mA} / \mathrm{V}
$$

Thus,

$$
r_{o}=\sqrt{\frac{500 \mathrm{k} \Omega}{2 \mathrm{~mA} / \mathrm{V}}}=\sqrt{250 \times 10^{6} \Omega^{2}}=15.81 \mathrm{k} \Omega
$$

and the required $V_{A}$ now becomes

$$
\begin{gathered}
r_{o}=V_{A} / I \quad \text { or, solving for } V_{A}, \quad V_{A}=I r_{o} \\
V_{A}=(0.200 \mathrm{~mA})(15.81 \mathrm{k} \Omega)=3.16 \mathrm{~V}
\end{gathered}
$$

which implies a channel length for each of the two transistors in the cascode of

$$
L=\frac{V_{A}}{V_{A}^{\prime}}=\frac{3.16 \mathrm{~V}}{5 \mathrm{~V} / \mu \mathrm{m}}=0.63 \mu \mathrm{~m}
$$

a considerable reduction from the case of a simple current source, and indeed a practical value.

