SINUSOIDAL STEADY-STATE ANALYSIS → FINDING THE PARTICULAR SOL$^{\underline{n}}$ WHEN THE FORCING FUNCTION IS SINUSOIDAL

I THE IMPORTANCE OF SINUSOIDS:

- ARISE IN THE NATURAL SOL$^{\underline{n}}$ OF RESONANT CIRCUITS (SYSTEMS)
- ELECTRICAL POWER IS DISTRIBUTED VIA SINUSOIDAL VOLTAGES
- SINUSOIDS OF DIFFERENT FREQ. ARE ORTHOGONAL

LET $x_1(t) = \cos(\omega_1 t + \phi_1)$, $x_2(t) = \cos(\omega_2(t) + \phi_2)$

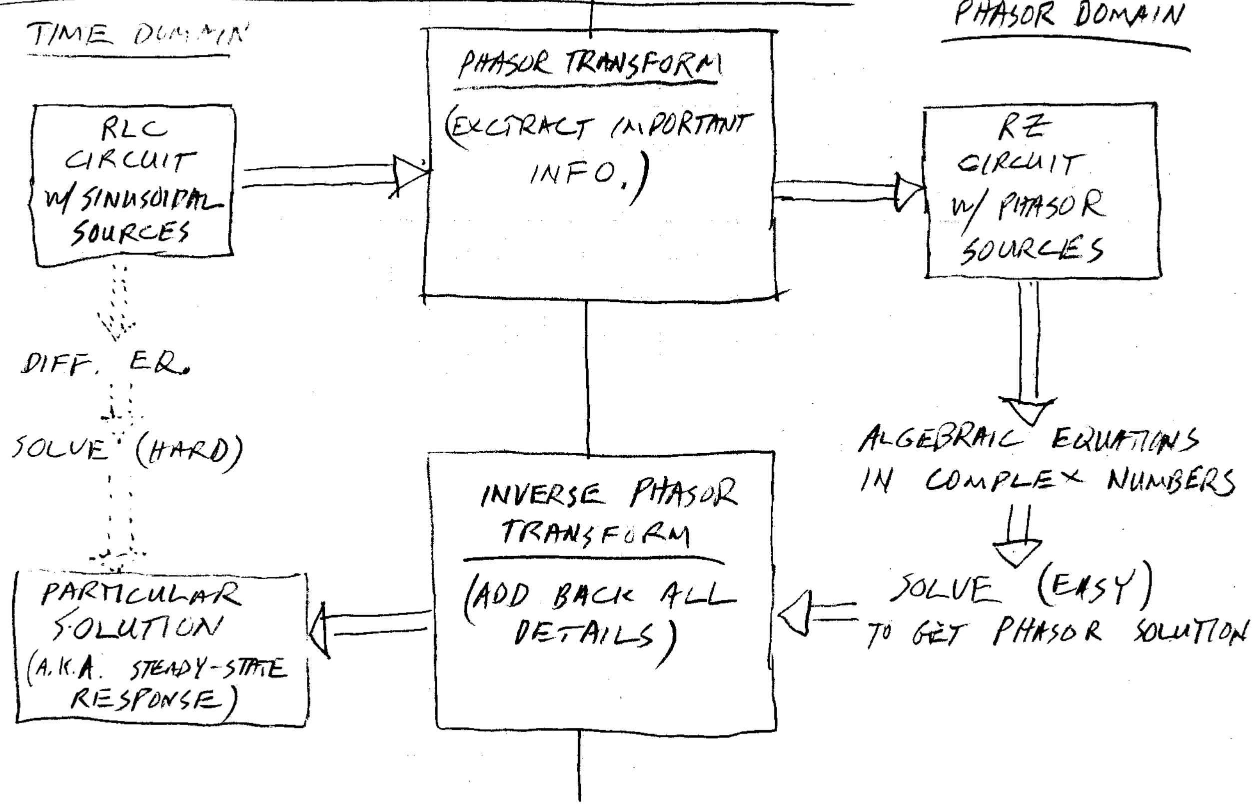
THEN $\lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x_1(t)\, x_2(t) = 0$ IF $\omega_1 \neq \omega_2$

(THERE ARE A NUMBER OF WAVEFORMS WITH THIS SPECIAL PROPERTY)

- BECAUSE OF ORTHOGONALITY, ANY PERIODIC WAVEFORM CAN BE CREATED AS A SUMMATION OF SINUSOIDS. (FOURIER ANALYSIS)

$\boxed{\text{SINUSOIDS ARE BASIC}}$

II A BIRD'S-EYE VIEW OF SINUSOIDAL ANALYSIS



TRANSFORM — A CHANGE IN MATHEMATICAL DESCRIPTION (USUALLY DONE TO FACILITATE SOLUTION OR TO GAIN INSIGHT FROM A NEW PERSPECTIVE)

III TRANSFORMING SINUSOIDS TO PHASORS

SUPPOSE $v(t) = V_m \cos(\omega t + \theta)$ ω IN RAD/SEC. θ IN RAD

OR $v(t) = V_m \cos\left(2\pi f + \frac{2\pi\phi}{360}\right)$ f IN Hz ϕ IN DEGREES

> A NOTE ON NOTATION: THE NOTATION $v(t) = V_m \cos(2\pi 60 t + 30^\circ)$ IS COMMON. IT SHOULD BE UNDERSTOOD AS
>
> $$v(t) = V_m \cos\left(120\pi t + \frac{2\pi \cdot 30}{360}\right)$$
>
> THE NOTATION OF "30°" FOR $\frac{2\pi \cdot 30}{360}$ IS A COMMON SHORT-HAND (SLANG)

- USUALLY ω OR f IS KNOWN FROM CONTEXT

  e.g. KDCR IS 88.5 MHz (f)

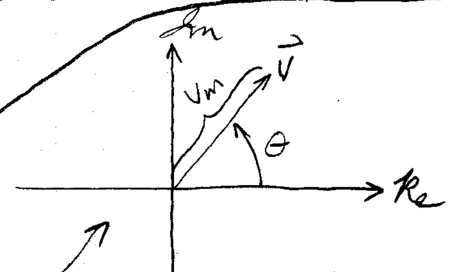
  e.g. AC POWER IN THE U.S. IS 60 Hz (f)

- AND THE FORCED RESPONCE ALWAYS HAS THE SAME FREQ. AS THE FORCING FUNCTION

- THEREFORE NO NEED TO SOLVE FOR f OR ω

- THE ONLY INTERESING NUMBERS NEEDED FOR THE SOLUTION ARE AMPLITUDE AND PHASE

- CONSIDER THEM TO FORM A COMPLEX NUMBER



IF $v(t) = V_m \cos \omega t + \theta$

WE CAN WRITE $\vec{V} = V_m \angle \theta$ AND UNDERSTAND WHAT IN MEANS.

THIS IS THE PHASOR TRANSFORM OF A SIGNAL

TIME DOMAIN PHASOR DOMAIN

PHASOR TRANSFORM

$i(t) = I_m \cos(\omega t + \theta)$ $\vec{I} = I_m \angle \theta$

INVERSE PHASOR TRANSFORM

NOTE: $i(t)$ IS A FUNCTION (OF TIME)

NOTE $\vec{I}$ IS A COMPLEX NUMBER

INTERLUDE — REVIEW COMPLEX NUMBERS

IV APPLICATION OF EULER'S RULE TO PHASORS

$$v(t) = V_m \cos(\omega t + \theta)$$

$$= V_m \mathrm{Re}\{\cos \omega t + \theta + j \sin \omega t + \theta\}$$

$$= V_m \mathrm{Re}\{e^{j(\omega t + \theta)}\}$$

$$= V_m \mathrm{Re}\{e^{j\theta}\} \mathrm{Re}\{e^{j\omega t}\}$$

TAKE PHASOR TRANSFORM (DIVIDE THRU BY $\mathrm{Re}\{e^{j\omega t}\}$ AND $j\,\mathrm{Im}\{e^{j\theta}\}$)

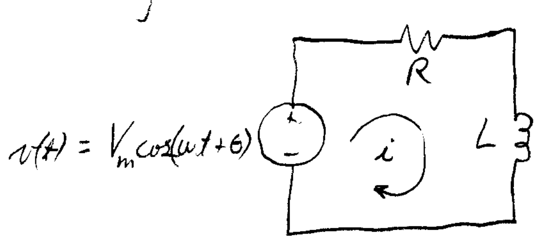
$$\bar{V} = V_m e^{j\theta}$$

SCALING ADDITIVITY

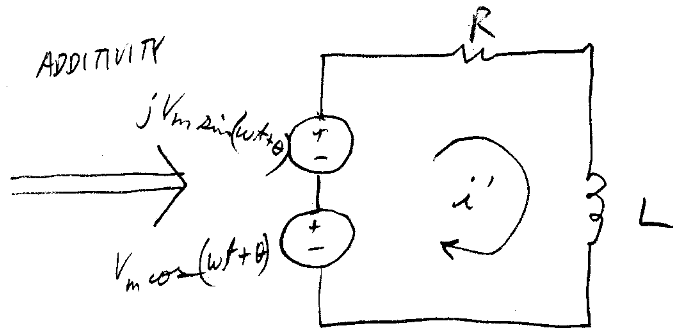
$$\bar{V} = V_m \angle \theta$$

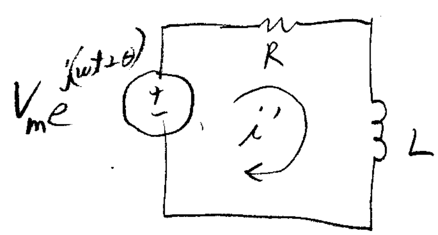
∴ BY SUPERPOSITION, WE OUGHT TO BE ABLE TO SOLVE CCT PROBLEMS USING PHASORS RATHER THAN FUNCTIONS

eg FIND i'



ADDITIVITY





$$V_m e^{j(\omega t + \theta)} = R i' + L \frac{di'}{dt}$$

ASSUME $i'(t) = I_m e^{j(\omega t + \phi)}$

$$\frac{di'}{dt} = j\omega I_m e^{j(\omega t + \phi)}$$

$$V_m e^{j(\omega t + \theta)} = R I_m e^{j(\omega t + \phi)} + L j\omega I_m e^{j\omega t + \phi}$$

$$V_m e^{j\theta} = R I_m e^{j\phi} + j\omega L I_m e^{j\phi}$$

$$V_m e^{j\theta} = (R + j\omega L) I_m e^{j\phi}$$

$$I_m e^{j\phi} = \frac{V_m e^{j\theta}}{(R + j\omega L)}$$

THE PROCESS IS JUST LIKE DC ANALYSIS! (ONLY USING COMPLEX NUMBERS)

V APPLICATION OF PHASOR CONCEPT TO SIMPLE CIRCUIT ELEMENTS

A.) FOR AN INDUCTOR

TIME DOMAIN → PHASOR DOMAIN

LET $i = I_m \cos(\omega t + \theta) \longrightarrow$ LET $\bar{I} = I_m \angle\theta \quad (= I_m e^{j\theta})$

$$v = L\frac{di}{dt}$$

$$v = -I_m L\omega \sin(\omega t + \theta)$$

$$v = I_m L\omega \cos(\omega t + \theta + 90^\circ) \longrightarrow \bar{V} = I_m L\omega \angle \theta + 90^\circ$$

$$\bar{V} = (I_m \angle\theta)(\omega L \angle 90^\circ)$$

$$\bar{V} = (I_m \angle\theta)(j\omega L)$$

$$\boxed{\therefore \frac{\bar{V}}{\bar{I}} = j\omega L \text{ FOR AN INDUCTOR}}$$

B) FOR A CAPACITOR

LET $v = V_m \cos(\omega t + \theta) \longrightarrow \bar{V} = V_m \angle\theta \quad (= V_m e^{j\theta})$

$$i = C\frac{dv}{dt}$$

$$i = -V_m C\omega \sin(\omega t + \theta)$$

$$i = V_m C\omega \cos(\omega t + \theta + 90^\circ) \longrightarrow \bar{I} = V_m C\omega \angle \theta + 90^\circ$$

$$\bar{I} = (V_m \angle\theta)(C\omega \angle 90^\circ)$$

$$\bar{I} = (V_m \angle\theta)(j\omega C)$$

$$\boxed{\therefore \frac{\bar{V}}{\bar{I}} = \frac{1}{j\omega C} \text{ FOR A CAPACITOR}}$$

C) FOR A RESISTOR

LET $i = I_m \cos(\omega t + \theta) \longrightarrow \bar{I} = I_m \angle\theta$

$$v = R I_m \cos(\omega t + \theta) \qquad \bar{V} = R I_m \angle\theta$$

$$\boxed{\therefore \frac{\bar{V}}{\bar{I}} = R}$$

D) DEFINITION OF IMPEDANCE (AND ADMITTANCE)

RESISTANCE IS THE RATIO $R = \frac{V}{I}$.

IF WE GENERALIZE THIS TO PHASORS, WE HAVE IMPEDANCE.

defn IMPEDANCE IS THE RATIO OF PHASORS $\bar{Z} = \frac{\bar{V}}{\bar{I}}$

ALTHOUGH $\bar{Z}$ IS A COMPLEX NUMBER IT IS NOT A PHASOR BECAUSE IT DOES NOT REPRESENT A SINUSOID.

THE IMPEDANCE OF SIMPLE CIRCUIT ELEMENTS IS:

RESISTOR:	$\bar{Z} = R$
INDUCTOR	$\bar{Z} = j\omega L$
CAPACITOR	$\bar{Z} = \frac{1}{j\omega C}$

NOTE THAT IN GENERAL, IMPEDANCE IS A FUNCTION OF FREQUENCY (ω)

IMPEDANCE HAS UNITS OF OHMS

JUST AS THE RECIPROCAL OF RESISTACE IS CONDUCTANCE (IN SIEMENS)
THE RECIPROCAL OF IMPEDANCE IS ADMITTANCE (ALSO IN SIEMENS)

defn $G = \frac{1}{R} \longrightarrow \bar{Y} = \frac{1}{\bar{Z}} = \frac{\bar{I}}{\bar{V}}$

E) OBSERVATIONS ABOUT IMPEDANCE (AND ADMITTANCE)

- IMPEDANCE IS A COMPLEX NUMBER. IT IS NOT A PHASOR BECAUSE IT CAN'T BE TRANSFORMED BACK TO THE TIME DOMAIN
IT DOES NOT REPRESENT A SINUSOID.

- IMPEDANCE IS A COMPLEX NUMBER, IT HAS A REAL AND AN IMAGINARY PART

observation: .... → $\mathrm{Re}\{\bar{Z}\} = R$ RESISTANCE (IN OHMS)

defn .... → $\mathrm{Im}\{\bar{Z}\} = X$ REACTANCE (IN OHMS)

$\bar{Z} = R + jX$ WHERE $R \in$ {REAL NUMBERS}
$X \in$ {REAL NUMBERS}

FOR A CAPACITOR $\bar{Z}_c = \frac{1}{j\omega C}$ $R_c = 0$, $X_C = -\frac{1}{\omega C}$

$$\bar{Z}_C = 0 + j\left(-\frac{1}{\omega C}\right) = \frac{j}{j} \cdot j \cdot \frac{-1}{\omega C} = \frac{(-1)(-1)}{j\omega C} = \frac{1}{j\omega C}$$

FOR AN INDUCTOR $\bar{Z}_L = j\omega L$ $R_L = 0$ $X_L = \omega L$

$$\bar{Z}_L = 0 + j(\omega L)$$

SIMILARLY FOR ADMITANCE, $\bar{Y} = \frac{1}{\bar{Z}}$

defn

$\mathrm{Re}\{\bar{Y}\} = G$ CONDUCTANCE (IN SIEMENS)

$\mathrm{Im}\{\bar{Y}\} = B$ SUSCEPTANCE (IN SIEMANS)

$\bar{Y} = G + jB$

FOR A CAPACITOR $\bar{Y} = j\omega C$ $G = 0$, $B = \omega C$

FOR AN INDUCTOR $\bar{Y} = \frac{1}{j\omega L}$ $G = 0$, $B = -\frac{1}{\omega L}$

$\bar{Y} = \frac{1}{\bar{Z}}$ BUT $G \neq \frac{1}{R}$ $\left(G = \frac{1}{R} \text{ ONLY IF } X = 0\right)$

AND $B \neq \frac{1}{X}$

WE HAVE $\bar{Y} = \frac{1}{\bar{Z}}$

$$G + jB = \frac{1}{R + jX}$$

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

$$\boxed{\therefore G = \frac{R}{R^2 + X^2} \qquad B = \frac{-X}{R^2 + X^2}}$$

DEFN OF CONDUCTANCE IN PHASOR DOMAIN

WE SAY AN IMPEDANCE IS PURELY RESISTIVE IF $X = 0$ (A RESISTOR)
" " REACTIVE IF $R = 0$ (INDUCTOR OR CAPACITOR)
ADMITANCE " CONDUCTIVE IF $B = 0$ (CONDUCTOR)
" " SUSCEPIVE IF $G = 0$ (CAPACITOR OR INDUCTOR)