

Suggested directions of thinking for answering problem 3.19 on EGR 304 PS #6.

In the grand tradition of courses on probability and stochastic systems, many homework problems involve planes and trains. This example is about a train. Please re-interpret it for the problem at hand.

The situation:

Suppose a commuter train travels around a loop of track with stations spaced along the loop. At each station the train stops for exactly 2 minutes for passengers to exit and enter. Then it resumes travel and proceeds to the next station. There are 20 stations on the loop. It takes 5 minutes (exactly) for the train to move from one station to the next. Thus, as the train travels around the loop exactly once it spends 40 minutes waiting at stations and 100 minutes running on the tracks for a total cycle time of 140 minutes. The train always goes around the loop in the same direction in this pattern all day every day.

Workers need to repair the tracks occasionally. They do not want to have a train go over any part the segment of track upon which they are working because it is an electrically powered train and they want the entire segment de-energized. The workers can see the stations at each end of the segment of track they are working on and they can see all the track in the segment needing work, but they cannot see beyond those two stations. The workers also cannot see inside the station to observe if a train is waiting, thus, even though they can see that the track is clear for the entire segment, a train might exit the station upstream from them at any moment. (IRQ!) If there is no train in sight they are assured the track is de-energized. The repair work, by an amazing coincidence, takes exactly 2 minutes to complete.

The workers arrive at a segment of track between two stations at a random moment in time. If they observe that the train is not on their segment of track, they commence work immediately. This work is "safety critical" meaning that if the train enters the segment upon which work is being done, "something bad will happen." If the train's timing cannot be altered to accommodate the work and the work starts as described, what is the chance that, "something bad will happen?"

Analysis:

If there is no train in the station upstream from their work when they commence work, they are OK. Even if a train arrives in the station after they commence work, the work will be finished before the train will exit the station so the track will remain de-energized for the duration of their work.

Thus, I'm interested to know if a train is in the upstream station when work commences. If the train is in that station it will exit before the work finishes and, "something bad will happen." The train, if it is in the station, will occupy the station for 2 minutes out of a possible 135 minutes of the cycle. (The total cycle is 140 minutes, but it is known that the train is not on the segment to be worked on so that's 5 minutes out of the total cycle.) There are 135 minutes of track and station travel in play but only 2 of those minutes relate to the one upstream station of concern. The odds of trouble are $2/135$ meaning that there is a 1.48% chance that the train is in the station when work starts, and that means trouble!

More scenario:

The track happens to break down frequently. On average, it requires 12 repairs per 24-hour day! (Thus on average there will be two hours between repairs.) What is the average time between those repairs that cause trouble? This is the same as asking how often a troublesome repair is likely to occur.

More Analysis: Over a 1000-day interval there will be on average 12000 repairs. Only $2/135 = 1.48\%$ of them cause trouble. Thus over a 1000-day interval $12000(2/135) = 177.8$ of those repairs cause trouble in the 1000-day or 24000-hour interval. $24000/177.8 = 135$ hours between troublesome repairs. (Or $1000/177.8 = 5.63$ days between troublesome repairs, on average.)

Still more scenario:

What are the chances of going 5 sequential days without a troublesome repair?

Still more analysis:

In 5 days there will be $12 \times 7 = 60$ repairs. Each repair has a $(1 - 2/135) = 133/135$ chance of being trouble free. The chance of 60 such repairs in a row being trouble-free is $(133/135)^{60} = 40.8\%$ (Even though 5.63 days between trouble on average, one will not so often see 5 days in a row without trouble! Counter-intuitive isn't it.)