1. Plot the following eight numbers as vectors in the complex plane: $z_1 = 3$, $z_2 = 1\angle\pi$, $z_3 = 1\angle\pi/2$, $z_4 = -2j$, $z_5 = 3e^{j\pi/6}$, $z_6 = 1 - j$, $z_7 = 2e^{j3\pi/4}$, $z_8 = 3\angle -510°$.
2. Express each of the following eight numbers in (a) exponential form and (b) polar form. (Write the argument of each number, θ, so that $-\pi < \theta \leq \pi$.) $z_1 = 1$, $z_2 = j$, $z_3 = -1$, $z_4 = -j$, $z_5 = 1 + j$, $z_6 = 1 - j$, $z_7 = -1 - j$, $z_8 = -1 + j$.
3. Express each of the following seven numbers in rectangular form: $z_1 = 1\angle 0°$, $z_2 = 2e^{j\pi}$, $z_3 = 3e^{-j\pi/2}$, $z_4 = 10\angle -450°$, $z_5 = -3e^{j\pi/4}$, $z_6 = 7\angle 30°$, $z_7 = -(2\angle 60°)$.
4. (a) Illustrate $z_1 + z_2 + z_3$ graphically as the sum of vectors in the complex plane, where $z_1 = 1$, $z_2 = j$, and $z_3 = 2\angle 135°$.
   (b) Find $|z_1 + z_2 + z_3|$ and $\arg\{z_1 + z_2 + z_3\}$.
5. Draw z_1, z_2, and z_1z_2 as vectors in the complex plane, where $z_1 = 2\angle 30°$ and $z_2 = -4 + 4j$. Specify both the polar and the rectangular coordinates of z_1z_2.
6. Draw z as an arbitrary vector in the complex plane, showing its length $|z|$ and angle θ and its rectangular coordinates x and y. Draw $1/z$ on the same plane and indicate its length and angle and its rectangular coordinates.
7. Evaluate each of the following expressions, putting your answers in (i) rectangular form, (ii) exponential form, and (iii) polar form.

   (a) $\dfrac{3\angle 60°}{1 + 3j}$ (b) $\dfrac{1 + j}{-1 - 3j}$

   (c) $\dfrac{e^{2j} + 1}{e^{j} - 1}$ (d) $\dfrac{e^{2j} + 1}{e^{j} + 1}$

   (e) $\dfrac{(1 - j)(4e^{j45°})}{1 + 3j + 3\angle 60°}$

   (f) $\dfrac{2\angle 45° + 8\angle 135°}{(4\angle 90°)(2\angle 20°)}$

   (g) $(5\angle 22°)^{-1} + (6\angle 185°)^{-1}$

   (h) $\dfrac{4\angle 65° + (4 + j)^{-1}}{17 + e^{j60°}}$
8. Verify Eqs. (B.25) and (B.26).
9. Verify Eq. (B.27).
10. Verify Eqs. (B.28), (B.29), and (B.30).
11. Find the magnitudes and angles of the following expressions. (All quantities are real except j.)

    (a) $\dfrac{1}{1 + j\omega RC}$ (b) $\dfrac{V_m\angle\phi}{j\omega RC + 1}$

    (c) $\dfrac{1 + j\omega RC}{R - \omega^2 RLC + j\omega L}$ (d) $\dfrac{(a + bj)(c + dj)}{(e + fj)(h + ij)}$
12. Find the real and the imaginary parts of each of the following expressions. Put your answers in the simplest form possible. (All quantities are real except j.)

    (a) $\dfrac{V_m\angle\phi}{1 + j\omega RC}\, e^{j\omega t}$

    (b) $\dfrac{(1 + j\omega RC)V_m e^{j\phi}}{R - \omega^2 RLC + j\omega L}\, e^{j\omega t}$

    (c) $\dfrac{j\omega_1 L(A_1\angle\phi_1)}{2j\omega_1 L + R}\, e^{j\omega_1 t} + \dfrac{j\omega_2 L(A_2\angle\phi_2)}{2j\omega_2 L + R}\, e^{j\omega_2 t}$
13. Find the roots of the following equations and plot them as points in the complex plane.
    (a) $s^2 + 4s + 8 = 0$
    (b) $s^2 + 6s + 8 = 0$
    (c) $s^8 = 1$
    (d) $s^7 = 1$
14. Shade the regions in the complex (z) plane that satisfy
    (a) $\mathcal{R}e\,\{z\} \geq 1$
    (b) $\mathcal{I}m\,\{z\} \geq 1$
    (c) $|z| \leq 1$
    (d) $|z - 1| = 3$
15. Under what conditions does $|z_1 + z_2| = |z_1| + |z_2|$?
16. Under what conditions does $|z_1 + z_2| = |z_1| - |z_2|$?
17. (a) Take the real part of $e^{j(\theta + \phi)}$ to show that $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$.
    (b) Take the imaginary part of $e^{j(\theta+\phi)}$ to show that $\sin(\theta + \phi) = \cos\theta\sin\phi + \sin\theta\cos\phi$.
18. Show that $z_1z_2 = 0$ if and only if at least one of the numbers z_1, z_2 is zero.
19. Show that

    $$|z_1 + z_2 + \cdots + z_n| \leq |z_1| + |z_2| + \cdots + |z_n|$$

    When does the equality hold?
20. Let n be any integer. Express the nth roots of unity in (a) exponential form and (b) rectangular form.
21. Evaluate each of the following expressions, putting your answers in (i) rectangular form, (ii) exponential form, and (iii) polar form.
    (a) $j^{\sqrt{j}}$
    (b) j^{e}
    (c) $\cos(3 + 2j)$
    (d) $\sin(3 + 2j)$
    (e) $1 + z + z^2 + \cdots$, where $|z| < 1$. (*Hint*: Let S equal the infinite series and subtract zS from S to get a closed form for S.)
    (f) $1 + z + z^2 + \cdots + z^N$, where N is an integer.
22. Let $V(s) = 1/(s - s_0)$ where s and s_0 are complex numbers. Show that, in general, $V^*(s) \neq V(s^*) \neq V^*(s^*)$.